Van Driest Generalization Applied to Turbulent Skin Friction and Velocity Profiles Measured on the Wall of a Mach 7.4 Wind Tunnel

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Introduction

THE Van Driest mixing length method[1,2] predicts within 10% the measured turbulent skin friction and boundary-layer velocity profiles for sharp-edged flat plates at Mach numbers up to 8 (Refs. 3–5). In this Note, the Van Driest equations are presented in a more general form and applied to measurements of boundary-layer profiles on the test section wall of a Mach 7.4 wind tunnel,[6] for which the temperature-velocity distributions are different from the Crocco distribution. These data were compared previously[6] to the predictions of a finite-difference method.

Van Driest Equations for a Non-Crocco Temperature Profile

In a previous study of flat-plate results,[4] the Van Driest transformation functions for a Crocco temperature profile (for unit Prandtl number, $Pr = 1$) were listed for the following incompressible relationships:

$$\bar{U}/\bar{U}_\tau = f_1(\bar{U}_\tau \bar{y}/\bar{v}) \tag{1}$$

$$(\bar{U} - \bar{U}_e)/\bar{U}_\tau = f_2(\bar{y}/\bar{\delta}) \tag{2}$$

$$\bar{C}_f = f_3(\bar{R}_\theta) \tag{3}$$

where U is velocity; $(\ )_e$, boundary-layer edge condition; $U_\tau = (\tau_w/\rho_w)^{1/2}$, friction velocity; τ_w, wall shear stress; ρ_w, wall mass density; y, distance normal to surface; δ, boundary-layer thickness; v, kinematic viscosity; and C_f, local skin-friction coefficient. Equation (1) is the law of the wall for the inner part of the boundary layer and Eq. (2) is the velocity-defect law for the outer part. In Ref. 4 and herein, Coles' empirical incompressible curves[7] are used to represent the incompressible functions in Eqs. (1) and (2) and the Kármán-Schoenherr skin-friction formula is used with Eq. (3).

For this study, the Van Driest functions[4] were rewritten in terms of the static-temperature ratio, T_e/T, as follows:

$$\bar{U}/\bar{U}_\tau = (2/C_f)^{1/2} \int_0^{U/U_e} (T_e/T)^{1/2}\, d(U/U_e) \tag{4}$$

$$\bar{U}_\tau \bar{y}/\bar{v} = (\tau_w/\rho_w)^{1/2} y/v_w \tag{5}$$

$$\bar{y}/\bar{\delta} = y/\delta \tag{6}$$

$$C_f/\bar{C}_f = \left[\int_0^1 (T_e/T)^{1/2}\, d(U/U_e) \right]^2 \tag{7}$$

$$R_\theta/\bar{R}_\theta = \mu_w/\mu_e \tag{8}$$

where μ is absolute viscosity. A Crocco temperature-velocity distribution in Eqs. (4) and (7) leads to the closed expressions listed in Ref. 4. A quadratic total temperature-velocity distribution, $T_t^* = (U/U_e)^2 = (T_t - T_w)/(T_{t,e} - T_w)$, leads to $T_e/T = \{[1-(T_w/T_e)](U/U_e)^2 + T_w/T_e\}^{-1}$ which also results in the closed expressions for Eqs. (4) and (7) as follows:

$$\bar{U}/\bar{U}_\tau = [2T_e/C_f(T_w - T_e)]^{1/2} \sin^{-1}\{(U/U_e)[1-(T_e/T_w)]^{1/2}\} \tag{9}$$

$$C_f/\bar{C}_f = \frac{\{\sin^{-1}[1-(T_e/T_w)]^{1/2}\}^2}{(T_w/T_e) - 1} \tag{10}$$

Experiment

The experiment[6] was conducted in air on the wall of the Ames 3.5-Foot Hypersonic Wind Tunnel equipped with the Mach 7.4 nozzle. A total-temperature rake, pitot-pressure rake, and skin-friction balance were located at a single station near the end of the test section about 10 m downstream of the wind-tunnel throat. Test results are listed in Ref. 6.

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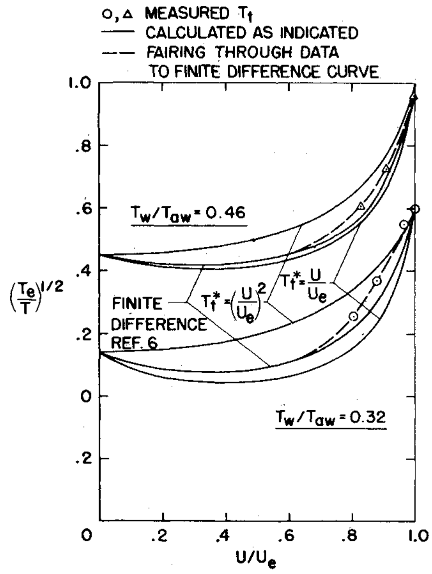

Fig. 1 Temperature-velocity distributions for Van Driest transformation functions [Eqs. (4) and (7)]; $M_e = 7.4$.

Temperature-Velocity Distributions

To examine the effect of temperature distribution on Eqs. (4) and (7), Fig. 1 presents $(T_e/T)^{1/2}$ vs U/U_e for two typical boundary-layer surveys.[6] (T_t^* vs U/U_e is shown in Ref. 6.) Even though the measurements span 80% of the boundary-layer thickness, it is apparent that insufficient data were taken near the wall. The measurements lie between the Crocco and quadratic curves, a result that was previously attributed to upstream flow-history effects from the strong pressure gradient near the tunnel throat.[8] Temperature distributions from a finite-difference method[6] lie between the measured distributions and the Crocco distribution. Areas under the curves indicate that the Van Driest integrals in Eqs. (4) and (7) will give considerably different results for each temperature distribution, the effects being greater at the lower T_w/T_{aw}. A representative experimental curve was

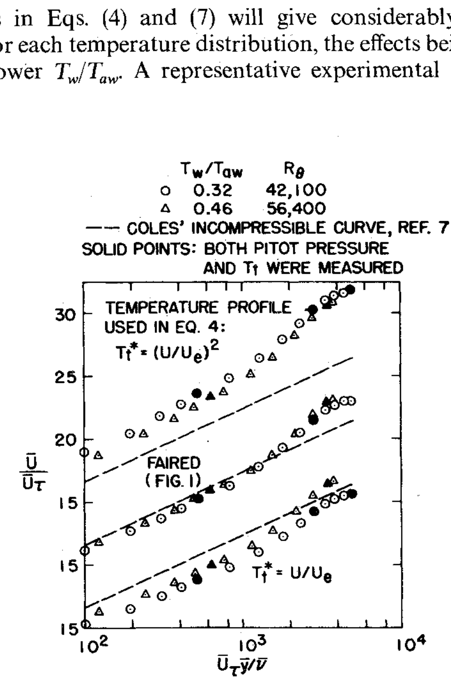

Fig. 2 Effect of changing the temperature-velocity distribution used in Eq. (4) on the law of the wall; $M_e = 7.4$.

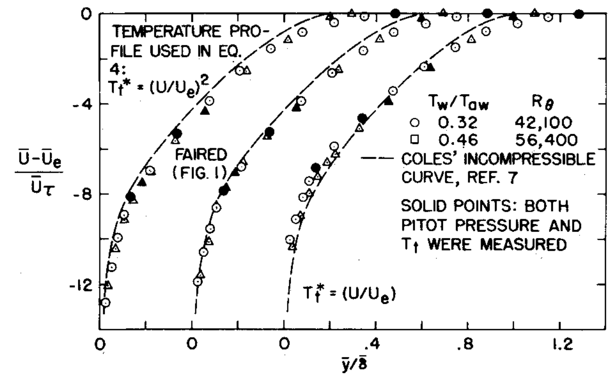

Fig. 3 Effect of changing the temperature-velocity distribution used in Eq. (4) on the velocity-defect law; $M_e = 7.4$.

obtained by fairing the measured temperatures to the finite-difference curve (dashed curve in Fig. 1). This faired curve, the Crocco and the quadratic representations were used in the figures that follow.

Velocity Profile Generalization

Figures 2 and 3 present the results from using Eqs. (4–6) with two velocity profiles from Ref. 6. Figure 2 shows that the assumed Crocco or quadratic temperature profiles do not transform the measured velocity profiles onto the Coles' incompressible curve. On the other hand, the faired experimental temperature distribution transforms the velocity profiles close to Coles' curve. In Fig. 3, the velocity-defect transformations lie close to the Coles' incompressible curve for all temperature distributions. The good correlation with Coles' curve in Fig. 2 obtained by using the modified Van Driest equations indicates that these equations can be applied to obtain skin friction from non-Crocco velocity profiles (Clauser technique) when $T(U)$ is known.

Skin-Friction Generalization

Figure 4 shows a comparison of the predicted and experimental skin friction using Eqs. (7) and (8). Measured R_θ was used in Eq. (8). Theoretical skin-friction values are greatly affected by the different temperature-velocity distributions, as shown by the large differences in the predicted skin friction. Using the faired experimental temperature-velocity curves, the skin friction is predicted to within about 3% of the measured values.

Thus, on the basis of these limited measurements, it appears that the Van Driest method can be extended to the prediction of non-Crocco type turbulent boundary-layer velocity profiles and skin friction by using a modified temperature-velocity distribution in the Van Driest equations.

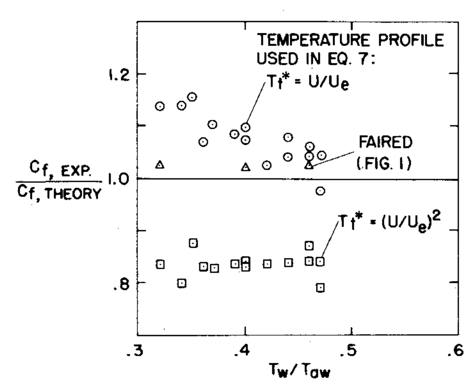

Fig. 4 Effect of changing the temperature-velocity distribution used in Eq. (7) on the prediction of skin friction; $M_e = 7.4$.

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Hypersonic Flight Results Showing Reynolds-Number Influence on Turbulent Base Pressure

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Nomenclature

A	= base area
D_s/D_b	= ratio of sting-to-model-base diameter
$\dot{m}$	= total heatshield ablation (mass-addition) rate
$\dot{m}/\rho_\infty V_\infty A$	= mass-addition parameter
M	= Mach number
p	= static pressure
R	= base radial coordinate
R_b, R_n	= base, nose radius
R_n/R_b	= bluntness ratio
Re	= Reynolds number
V	= velocity
θ_c	= cone half-angle
ρ	= static density

Subscripts

b	= base condition
e	= local cone (boundary-layer edge) condition immediately preceding base at L
L	= based on wetted length of cone
∞	= freestream condition

Introduction

THE base pressure of slender ($\theta_c < 15°$) conical re-entry bodies in turbulent flow at zero angle of attack may depend upon several variables: body geometry (θ_c, R_n/R_b, and

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